Solutions - Midterm Exam

(February 16th @ 7:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (20 PTS)

- Compute the result of the following operations. The operands are signed fixed-point numbers. The result must be a signed fixed-point number. For the division, use $x = 5$ fractional bits.

1.0111 + 1.101001	1.010101 − 1000.0101	01.11111 + 0.10001
10.101 × 1.01101	0.111 × 1.0101	10.101 ÷ 0.101

$c_8=1$ $c_7=1$ $c_6=1$ $c_5=1$ $c_3=0$ $c_2=0$ $c_1=0$ $c_0=0$

```
 1.0 1 1 1 0 0 +
 1.1 0 1 0 0 1
 -------------
 1.0 0 0 1 0 1
```

```
1 1 1 1.0 1 0 1 0 1 -
1 0 0 0.0 1 0 1 0 0
```

→

$c_{10}=1$ $c_9=1$ $c_8=1$ $c_7=1$ $c_6=1$ $c_5=1$ $c_4=1$ $c_3=1$ $c_2=0$ $c_1=0$ $c_0=0$

```
1 1 1 1.0 1 0 1 0 1 +
0 1 1 1.1 0 1 1 0 0
-------------------
0 1 1 1.0 0 0 0 0 1
```

$c_8=0$ $c_7=0$ $c_6=1$ $c_5=1$ $c_4=1$ $c_3=1$ $c_2=1$ $c_1=1$ $c_0=0$

```
0 0 1.1 1 1 1 1 +
0 0 0.1 0 0 0 1
---------------
0 1 0.1 0 0 0 0
```

```
10.101 x   →   01.011 x   →         1 0 0 1 1 x
1.01101        0.10011                1 0 1 1
                                    ---------
                                    1 0 0 1 1
                                  1 0 0 1 1
                                0 0 0 0 0
                              1 0 0 1 1
                            -----------------
                            0 1 1 0 1 0 0 0 1
                                    ↓
                            0.1 1 0 1 0 0 0 1
```

```
0.111 x   →   0.111 x   →        1 0 1 1 x
1.0101        0.1011               1 1 1
                                 -------
                                 1 0 1 1
                               1 0 1 1
                             1 0 1 1
                           ---------------
                           0 1 0 0 1 1 0 1
                                  ↓
                           0.1 0 0 1 1 0 1
                                  ↓
                           1.0 1 1 0 0 1 1
```

✓ $\frac{10.1010}{0.101}$: To unsigned (numerator) and then alignment, $a = 3$: $\frac{01.0110}{0.101} = \frac{01.011}{00.101} \equiv \frac{1011}{101}$

```
        001000110
101 ) 101100000
      101
      -----
        01000
          101
         ----
           110
           101
           ---
            10
```

Append $x = 5$ zeros: $\frac{101\mathbf{00000}}{101}$

Integer Division:

$Q = 1000110, R = 10$

$\rightarrow Qf = 10.00110\ (x = 5)$

Final result (2C): $\frac{10.1010}{0.101} = 2C(010.0011) = 101.1101$

PROBLEM 2 (10 PTS)

- Represent these numbers in Fixed Point Arithmetic (signed numbers). Use the FX format [12 4].

✓ -16.125

+16.125 = 010000.001 ⇒ -16.125 = 11101111.1110

✓ 19.25

+19.25 = 00010011.0100

- Complete the table for the following fixed point formats (signed numbers): (6 pts.)

Integer bits	Fractional Bits	FX Format	Range	Resolution
8	6	[14 6]	$[-2^7, 2^7 - 2^{-6}]$	2^{-6}
6	4	[10 4]	$[-2^5, 2^5 - 2^{-4}]$	2^{-4}

PROBLEM 3 (40 PTS)

- Perform the following 32-bit floating point operations. For fixed-point division, use 4 fractional bits. Truncate the result when required. Show your work: <u>how you got the significand and the biased exponent bits of the result</u>. Provide the 32-bit result.

✓ `40D00000 + C2EA0000`	✓ `50A90000 - 4F480000`	✓ `80200000 × 7AB80000`	✓ `FB380000 ÷ 48C00000`

✓ $X =$ `40D00000 + C2EA0000`:

`40D00000`: `0100 0000 1101 0000 1000 0000 0000 0000`

$e + bias = 10000001 = 129 \rightarrow e = 129 - 127 = 2$ $\quad Significand = 1.101$

`40D00000` $= 1.101 \times 2^2$

`C2EA0000`: `1100 0010 1110 1010 0000 0000 0000 0000`

$e + bias = 10000101 = 133 \rightarrow e = 133 - 127 = 6$ $\quad Significand = 1.110101$

`C2EA0000` $= -1.110101 \times 2^6$

$X = 1.101 \times 2^2 - 1.110101 \times 2^6 = +\frac{1.101}{2^4} \times 2^6 - 1.110101 \times 2^6 = (0.0001101 - 1.110101) \times 2^6$

To subtract these numbers, we first convert to 2C:

$R = 0.0001101 - 01.110101 = 0.0001101 + 10.001011$ (2C addition)

The result in 2C is: $R = 10.0100011$, $-R = 01.1011101$

* *Note that you can also do unsigned subtraction:* $X = -(1.110101 - 0.0001101 -) \times 2^6$

For floating point, we need to convert to sign-and-magnitude:

$\Rightarrow R(SM) = -1.1011101$

$c_9=0$	$c_8=0$	$c_7=0$	$c_6=0$	$c_5=1$	$c_4=1$	$c_3=1$	$c_2=0$	$c_1=0$	$c_0=0$	
	0	0.	0	0	0	1	1	0	1	+
	1	0.	0	0	1	0	1	1	0	
	1	0.	0	1	0	0	0	1	1	

$X = -1.1011101 \times 2^6$, $e + bias = 6 + 127 = 133 = 10000101$

X = `1100 0010 1101 1101 0000 0000 0000 0000` = `C2DD0000`

✓ $X =$ `50A90000 - 4F480000`:

`50A90000`: `0101 0000 1010 1001 0000 0000 0000 0000`

$e + bias = 10100001 = 161 \rightarrow e = 161 - 127 = 34$ $\quad Significand = 1.0101001$

`50A90000` $= 1.0101001 \times 2^{34}$

`4F480000`: `0100 1111 0100 1000 0000 0000 0000 0000`

$e + bias = 10011110 = 158 \rightarrow e = 158 - 127 = 31$ $\quad Significand = 1.1001$

`4F480000` $= 1.1001 \times 2^{31}$

$X = 1.0101001 \times 2^{34} - 1.1001 \times 2^{31} = 1.0101001 \times 2^{34} - \frac{1.1001}{2^3} \times 2^{34}$

$X = (1.0101001 - 0.0011001) \times 2^{34}$ (unsigned subtraction)

$b_8=0$	$b_7=0$	$b_6=0$	$b_5=1$	$b_4=0$	$b_3=0$	$b_2=0$	$b_1=0$	$b_0=0$	
	1.	0	1	0	1	0	0	1	-
	0.	0	0	1	1	0	0	1	
	1.	0	0	1	0	0	0	0	

$X = 1.001 \times 2^{34}$, $e + bias = 34 + 127 = 161 = 10100001$

X = `0101 0000 1001 0000 0000 0000 0000 0000` = `50900000`

✓ $X =$ `80200000 × 7AB80000`:

`80200000`: `1000 0000 0010 0000 0000 0000 0000 0000`

$e + bias = 00000000 = 0 \rightarrow Denormal\ number \rightarrow e = -126$ $\quad Significand = 0.01$

`80200000` $= -0.01 \times 2^{-126}$

`7AB80000`: `0111 1010 1011 1000 0000 0000 0000 0000`

$e + bias = 11110101 = 245 \rightarrow e = 245 - 127 = 118$ $\quad Significand = 1.0111$

`7AB80000` $= 1.0111 \times 2^{118}$

$X = (-0.01 \times 2^{-126}) \times (1.0111 \times 2^{118}) = -0.010111 \times 2^{-8} = -1.0111 \times 2^{-10}$

$e + bias = -10 + 127 = 117 = 01110101$

X = `1011 1010 1011 1000 0000 0000 0000 0000` = `BAB80000`

✓ $X =$ `FB380000 ÷ 48C00000`:

`FB380000`: `1111 1011 0011 1000 0000 0000 0000 0000`

$e + bias = 11110110 = 246 \rightarrow e = 246 - 127 = 119$ $\quad Significand = 1.0111$

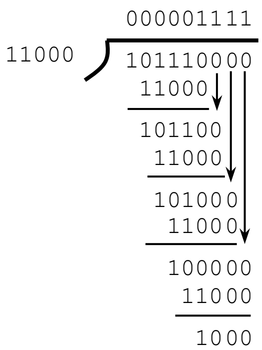
`FB380000` $= -1.0111 \times 2^{119}$

`48C00000`: `0100 1000 1100 0000 0000 0000 0000 0000`

$e + bias = 10010001 = 145 \rightarrow e = 145 - 127 = 18$ $\quad Significand = 1.1$

`48C00000` $= 1.1 \times 2^{18}$

$$X = -\frac{1.0111 \times 2^{119}}{1.1 \times 2^{18}} = -\frac{1.0111}{1.1} \times 2^{101}$$



Alignment:

$$\frac{1.0111}{1.1} = \frac{1.0111}{1.1000} = \frac{10111}{11000}$$

Append $x = 4$ zeros: $\frac{10111\mathbf{0000}}{11000}$

Integer division

$$Q = 1111 \rightarrow Qf = 0.1111$$

Thus: $X = -0.1111 \times 2^{101} = -1.111 \times 2^{100}$

$e + bias = 100 + 127 = 227 = 11100011$

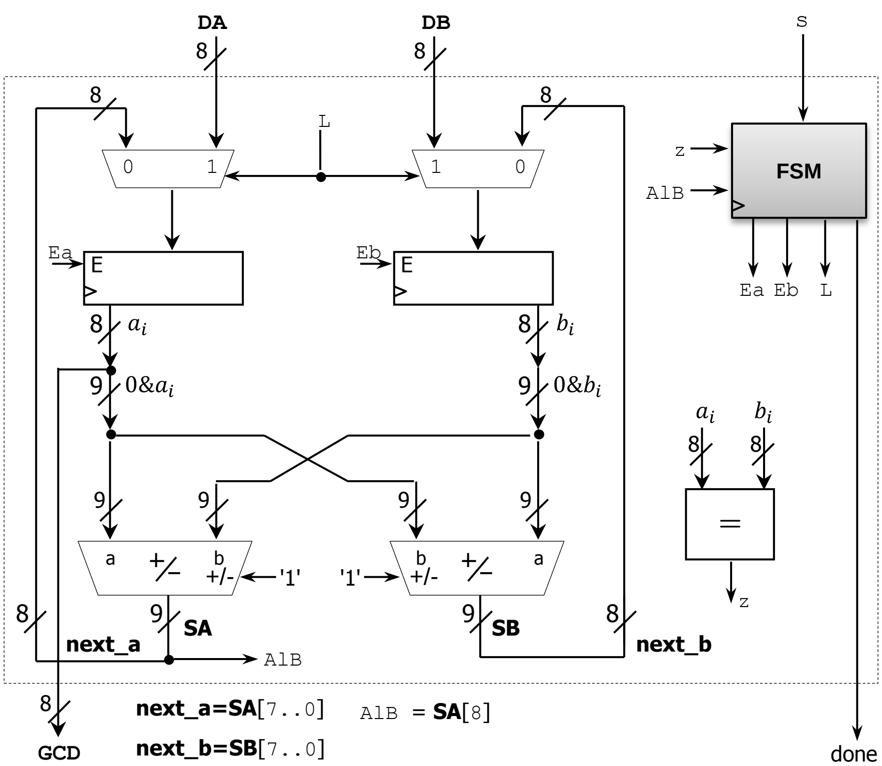
X = 1111 0001 1111 0000 0000 0000 0000 0000 = F1F00000

PROBLEM 4 (30 PTS)

- **Greatest Common Divisor** (GCD): This circuit computes the GCD of two n-bit unsigned numbers (A, B). For example:

✓ A = 216, B = 192 → GCD = 24.	✓ A = 132, B = 72 → GCD = 12.	✓ A = 169, B = 63 → GCD = 1.

- The digital system is depicted below (FSM + Datapath) for $n = 8$. This iterative circuit is based on Euclid's GCD algorithm.
  - ✓ Input Data: `DA`, `DB` Output data: `GCD`



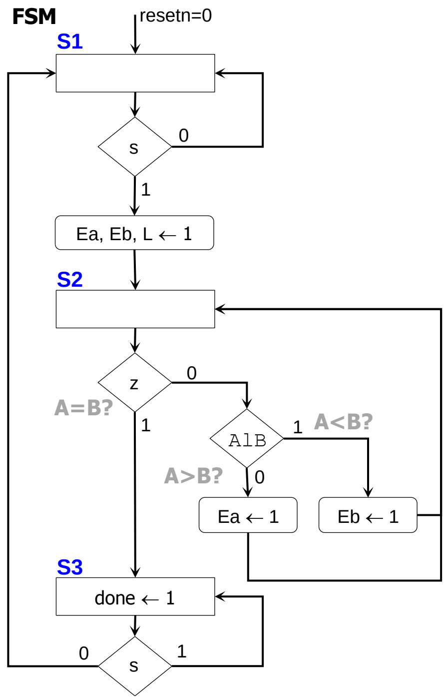
✓ z=1 when $a_i = b_i$, else z=0.

Sequential Algorithm

```
a,b: unsigned integers
while a ≠ b
    if a > b
        a ← a-b
    else
        b ← b-a
    end
end while
return a
```

- Sketch the Finite State Machine diagram (in ASM form) given the sequential algorithm (for $n = 8$). (18 pts.)
  - ✓ The process begins when s is asserted, at this moment we capture DA and DB on register a_i and b_i (respectively). Then the process continues by updating a_i and b_i and it is concluded when $a_i = b_i$. The signal done is asserted when the result is computed and appears on output GCD.

Finite State Machine:



- Complete the timing diagram where $n = 8$. DA and DB are provided as unsigned decimals. You can provide a_i and b_i as unsigned decimals. (12 pts.)